

Limitations and Losses of Conventional Tubes at Microwave Frequencies

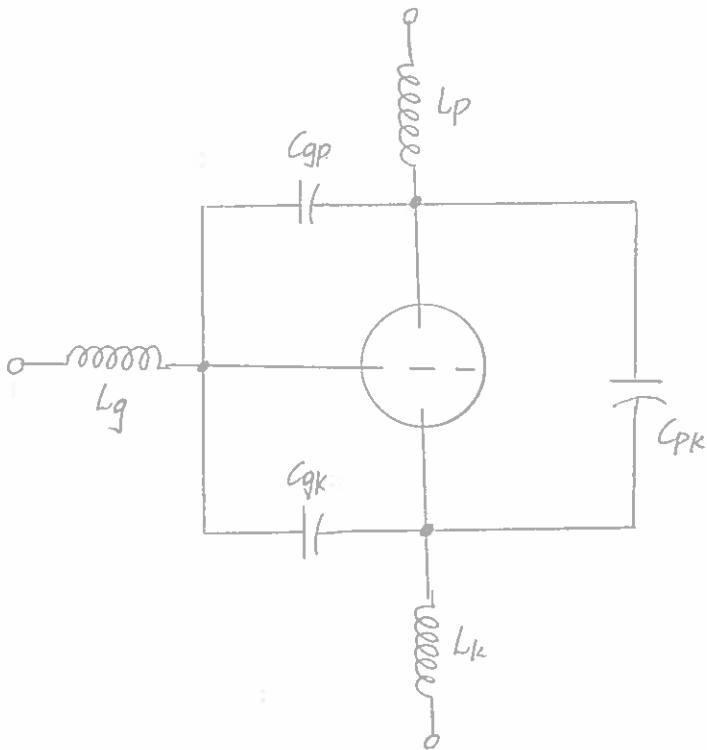
To see whether or not a conventional device like a triode or a transistor works satisfactorily at higher frequencies say UHF or at microwave frequencies, we can consider a simple oscillator and try to increase the operating frequency. In this process one would quite naturally proceed to reduce the tank circuit parameters i.e. either L or C . At this point, the device parameters like the inter-electrode capacitance and lead inductances are negligible. The circuit condition required for operation as an oscillator or as an amplifier may not be satisfied and the device at these frequencies becomes useless as an oscillator or as an amplifier. There are other reasons too i.e. conventional devices cannot be used for frequencies $> 100\text{MHz}$ because of the following effects.

- 1) Inter-electrode Capacitance effect
- 2) Lead inductance effect
- 3) Transit time effect
- 4) Grain bandwidth limitation
- 5) Effect due to RF losses
- 6) Effect due to radiation losses

Inter-electrode Capacitance Effect:

As frequency increases, the reactance $X_C = \frac{1}{2\pi f C}$ decreases and the output voltage decreases due to shunting effect. Because at higher frequencies X_C becomes almost a short. C_{gp} , C_{gk} and C_{pk} are the IEC's which come into effect.

The effect of IEC can be minimized by reducing the IEC's C_{gk} , C_{pk} and C_{gp} . These can be reduced by decreasing the area of the electrodes i.e. by using smaller electrodes or by increasing the distance between electrodes.



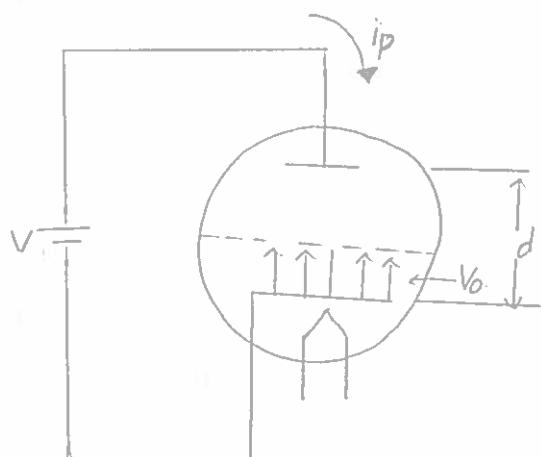
Lead Inductance Effect

As frequency increases, the reactance $X_L = 2\pi f L$, increases and hence the voltages appearing at the active electrodes are less than the voltages at the base pins. This results in reduced gain for the tube amplifier. L_k , L_p and L_g are the lead inductances that limit the performance of the tube.

The effect of L_k can be minimised by decreasing L . Since L is proportional to reactance, L can be decreased by using larger sized shield leads without base pins i.e. by increasing "A" and decreasing "l". This however reduces the power handling capability.

Transit time Effect

Transit time is the time taken for the electron to travel from cathode to anode.



$$\text{Transit time } \gamma = \frac{d}{V_0}$$

Where, d = distance b/w anode and cathode

V_0 = Velocity of electrons

Static energy of electrons = eV

$$\text{kinetic energy} = \frac{1}{2}mv_0^2$$

Under equilibrium, static energy = kinetic energy

$$eV = \frac{1}{2}mv_0^2$$

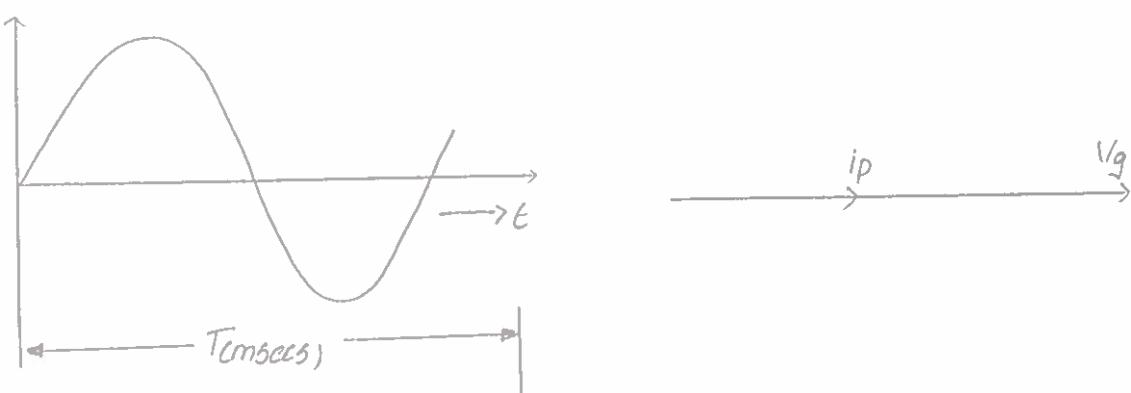
$$V_0 = \sqrt{\frac{2eV}{m}}$$

$$\gamma = \frac{d}{\sqrt{\frac{2eV}{m}}}$$

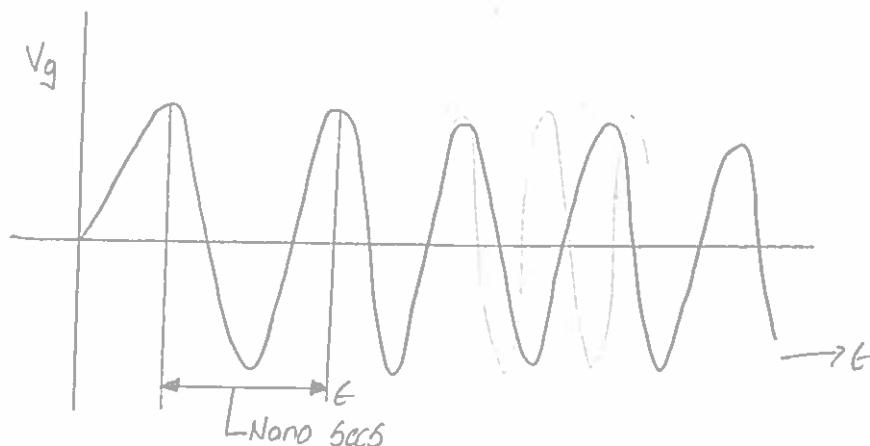
At low frequencies, transit time is negligible compared to the period of the signal.

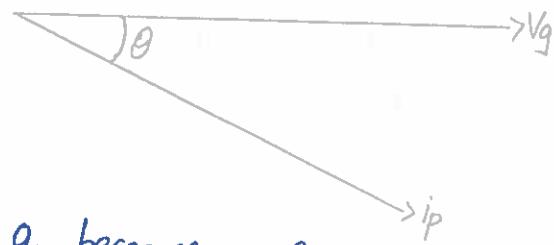
$$g_m = \frac{\Delta i_p}{\Delta V_g}$$

Both V_g and i_p are in phase. Therefore, the plate current i_p responds immediately or instantaneously to changes in Control grid Voltage V_g i.e. V_g and i_p are in phase.



At high frequencies the transit time γ is comparable with the period of the signal which is very small nano seconds.





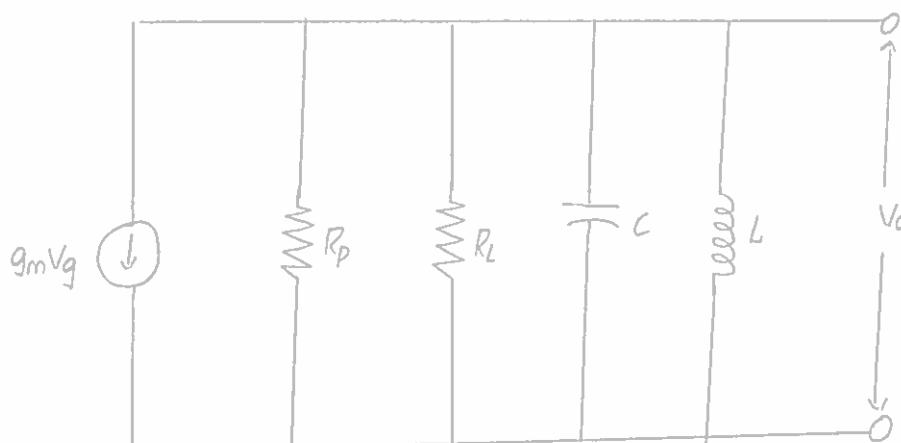
i_p lags V_g and g_m becomes a complex quantity. Therefore change in plate current occurs after a finite delay with respect to change in grid voltage V_g . Hence the plate current lags behind the control grid voltage. The gain then becomes complex even for a resistive load and real gain falls. It can be shown that $Y_{in} = k^2 R_k g_m C_{gk} + jk \omega C_{gk}$. This eqn shows that grid circuit absorbs power even if the grid is -ve w.r.t to Cathode and is proportional to the square of the frequency.

Gain Bandwidth Limitation

Maximum gain is achieved when the tuned circuit is at resonance. Referring to the equivalent circuit.

$$\text{Gain, } G_1 = \frac{V_o(s)}{V_i(s)}$$

$$= Z_0(s)$$



Equivalent Circuit

Applying Laplace transform to the parallel circuit and replacing R_L and R_p by

$$R = \frac{1}{R_L} + \frac{1}{R_p}$$

$$\frac{1}{Z_0(s)} = \frac{1}{V_o(s)} = Cs + \frac{1}{Ls} + \frac{1}{R} = \frac{s^2 LCR + Ls + R}{RLs}$$

$$Z_0(S) = \frac{S/C}{S^2 + \frac{S}{CR} + \frac{1}{LC}}$$

The characteristic eqn is given by the denominators $S^2 + \frac{1}{CR}S + \frac{1}{LC}$. The roots of the quadratic eqn give the cut-off frequencies ω_1 and ω_2 for calculating the bandwidth.

$$\omega_1 = \frac{G_1}{2C} - \sqrt{\left(\frac{G_1}{2C}\right)^2 - \frac{1}{LC}}$$

$$\omega_2 = -\frac{G_1}{2C} + \sqrt{\left(\frac{G_1}{2C}\right)^2 - \frac{1}{LC}}$$

Where $G_1 = \frac{1}{R}$

$$\text{Bandwidth, } BW = \omega_2 - \omega_1 = \frac{G_1}{C} \text{ for } \left(\frac{G_1}{2C}\right)^2 \gg \frac{1}{LC}$$

The maximum gain at resonance is $A_{max} = \frac{g_m}{G_1}$

$$\text{Gain Bandwidth product} = A_{max} \cdot BW = \frac{g_m}{G_1} \times \frac{G_1}{C} = \frac{g_m}{C}$$

The gain bandwidth product is thus independent of frequency. As g_m and C are fixed for a particular tube or circuit, higher gain can be achieved at the cost of bandwidth only. In microwave circuit, this restriction/limitation can be overcome

i) Resonant Cavities

ii) Slow Wave tubes

For a larger gain over a larger bandwidth.

Effect due to RF losses:

a) Skin effect losses:

These losses come into play at higher frequencies at which the current has the tendency to confine itself to a smaller cross section of the conductor towards its outer surface.

$$\delta = \text{Skin depth} = \sqrt{\sigma/2\pi f \mu_0}$$

$$\delta \propto \frac{1}{\sqrt{f}} \text{ and } \delta \propto A_{eff}$$

Where A_{eff} is the effective area over which current flows

$$A_{eff} \propto \frac{1}{f^p}$$

$$R = \frac{PQ}{A_{eff}}$$

$$R = \frac{PQ}{I^2 NF} = PQ JF$$

As F increases R increases.

Hence losses will increase at higher frequencies. These losses can be reduced by increasing the size of the conductors.

b) Dielectric losses:

This occurs in various types of insulating materials used in the device i.e. Spacers, glass envelope, Silicon or plastic encapsulation etc. The loss in any of these material is in general given by

$$P = \pi P V_0^2 E_0 \tan \delta$$

Where,

E_0 = relative permittivity of the dielectric

δ = loss angle of the dielectric

As P increases the power loss increases. The remedy for this is to eliminate the tube base and to reduce the surface area of glass.

Radiation Losses:

Whenever the dimensions of the tube approaches the wavelength, it will emit radiation that is, radiation losses increase with increase in frequency. The remedy for this is to use proper shielding of the tubes and its circuitry.

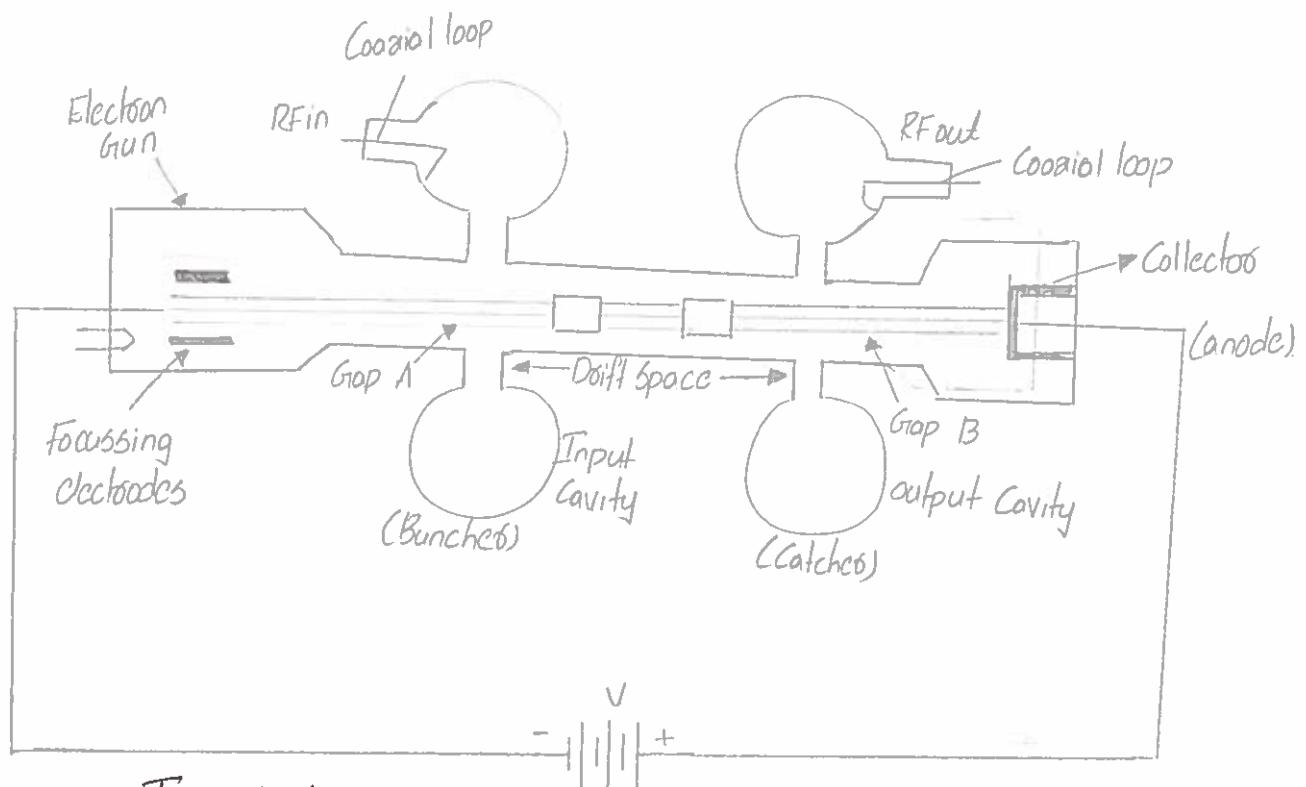
Microwave Tubes:

As already stated microwave tubes are constructed as to overcome the limitations of conventional and UHF tubes. They differ from them in that they make use of the transit time effect rather than fight it. In fact, large transit time is required for their operation. The basic principle of operation of the microwave tube involves transfer of power from a source of dc voltage to a source of dc voltage by means of current density modulated electron beam. The same is achieved by accelerating electrons in a static electric field and retarding them in an ac field. The density modulation of the electron beam allows more electrons to be retarded by ac field than accelerated by ac field which therefore makes possible a net

energy to be delivered to the ac electric field. The various types of microwave tubes that are available differ from each other in

- * Their mechanism of producing density modulation
- * The acceleration and retardation of electrons in an ac field
- * The retardation of electrons by a short gap or over an extended region

Two Cavity klystron Amplifier:

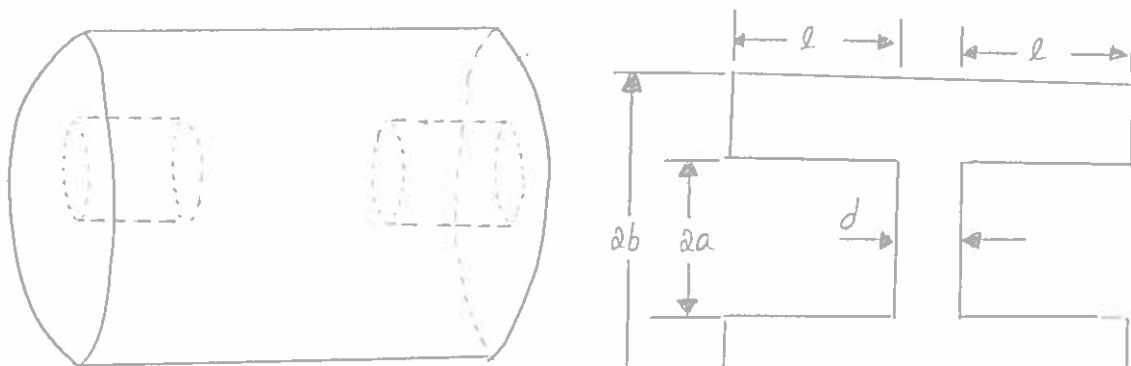


Two Cavity klystron Amplifier

- 1) A two cavity klystron amplifier which is basically a velocity modulated tube.
- 2) Here a high velocity electron beam is formed, focussed and sent down along a glass tube through an input cavity, a field free drift space and an output cavity to a collector electrode/anode.
- 3) The anode is kept at a positive potential with respect to Cathode.
- 4) The electron beam passes through a gap A consisting of two gaps of the buncher cavity separated by a very small distance and two other gaps of the catcher cavity with a small gap B.
- 5) The input and output are taken from the tube via resonant cavities with the aid of coupling loops.

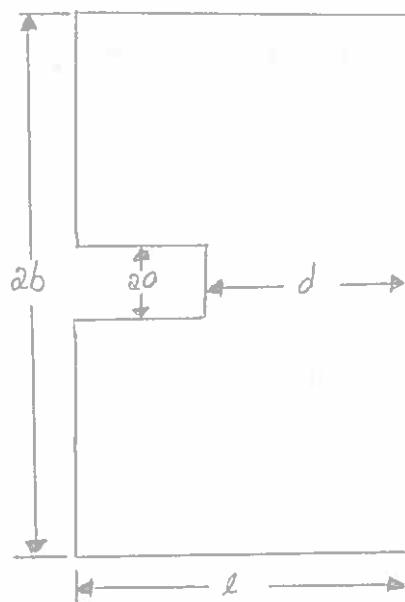
Resonant Cavities

For a Cavity resonator at microwave frequency, it is necessary that the inductance and Capacitance have to be Considerably reduced so that it maintains resonance at the operating frequency. Such a Cavity resonator whose the metallic boundaries Will extend into the interior of the Cavity are called oc-enhoant Cavities as in the Case of Coaxial Cavity.



Coaxial Cavity

Resonant Cavity Similar to a Coaxial line shorted at two ends and joined the Centre by a Capacitors. Such a resonant Cavity Can support an infinite number of resonant Frequencies. Hence it is Useful for making klystron devices. Another example of a resonant Cavity is the radial resonant Cavity.



Radial Resonant Cavity

In a resonant Cavity the inductance and resistance are reduced because of the hollow scoops Within. The Coaxial Cavity may be Considered as

a Coaxial line shorted at two ends RQ, RS and joined at the neck by a Capacitor.

Applying transmission line theory

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \text{ ohms}$$

The input impedance of the shorted line is

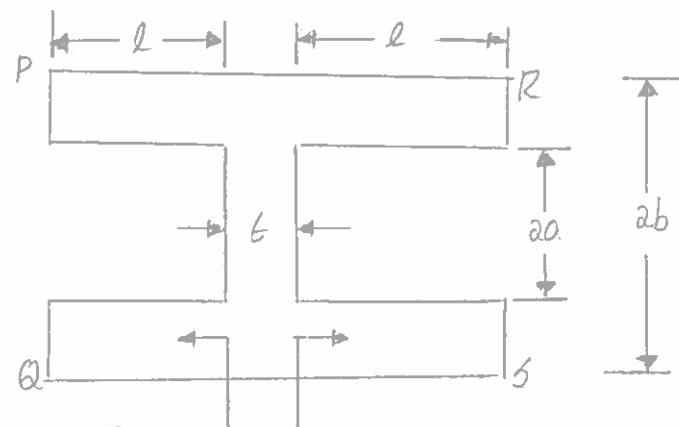
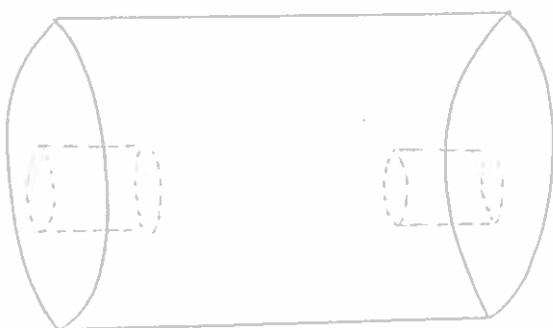
$$Z_{in} = j Z_0 \tan(BQ)$$

Where, l = length of Coaxial line

$$Z_{in} = j \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \tan(BQ)$$

The inductance L of the Cavity is given by

$$L = \frac{2Z_{in}}{BQ} = \frac{1}{\pi l Q} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \tan(BQ)$$



The Capacitance of the gap C_g is given by

$$C_g = \epsilon \frac{\pi a^2}{6}$$

$$\text{At resonance } LQ = \frac{1}{LC_g}$$

$$\frac{1}{\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \tan(BQ) = \frac{1}{l \epsilon \pi a^2}$$

$$\tan(BQ) = \frac{l}{la^2} \sqrt{\frac{1}{\mu \epsilon}} \times \frac{1}{\ln(b/a)}$$

A tangent function has infinite numbers of solutions and therefore, there will be infinite numbers of resonant frequencies or modes. The solution gives the resonant frequency of the Coaxial Cavity.

Velocity modulation process

Let the dc Voltage between Cathode and anode be V_0 and v_0 be the Velocity of the electron. L be the drift space length and the RF input signal to be amplified by the klystron be V_1 .

Then,

$$v_0 = \sqrt{\frac{2eV_0}{m}} \\ = 0.593 \times 10^6 \sqrt{V_0} \text{ m/sec}$$

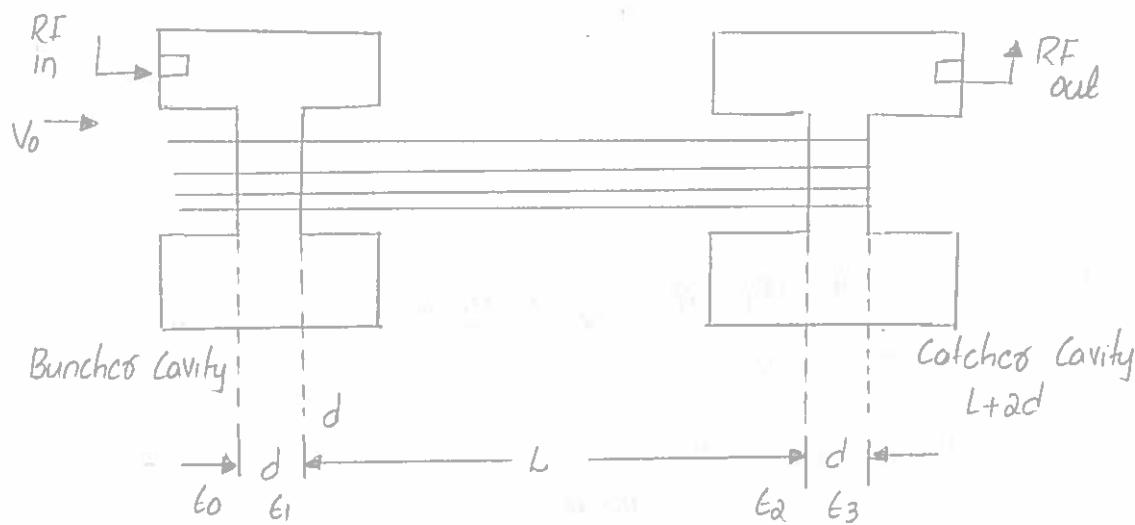
$$V_1 = V_1 \sin \omega t$$

Where,

V_1 = amplitude of the Signal and $V_1 \ll V_0$ is assumed.

The energy of the electron at the time of leaving buncher cavity is given by

$$\frac{1}{2}mv_0^2 = e(V_0 + V_1 \sin \omega t)$$



$$v_1 = \sqrt{\frac{2e(V_0 + V_1 \sin \omega t)}{m}}$$

$$= \sqrt{\frac{2eV_0}{m}} \cdot \sqrt{1 + \frac{V_1}{V_0} \sin \omega t}$$

Since $V_1 \ll V_0$

$$v_1 = v_0 \left[1 + \frac{V_1}{V_0} \sin \omega t \right]^{1/2}$$

Expanding binomially and neglecting higher powers of $\sin \omega t$, we get

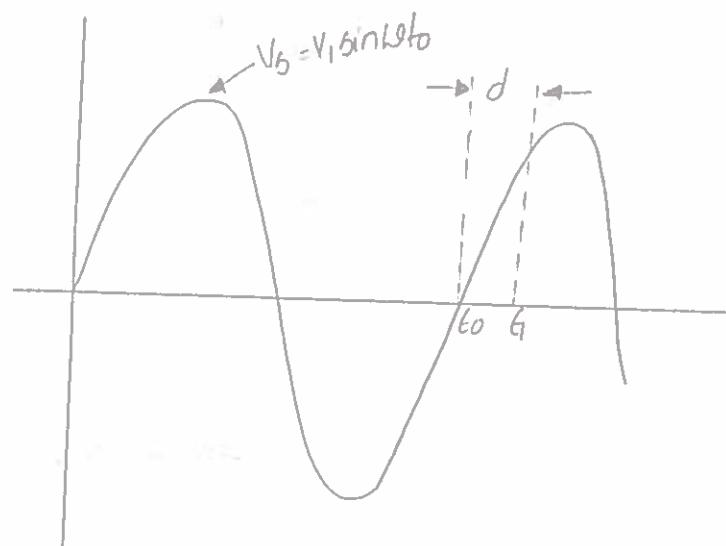
$$\theta_t = \theta_0 \left[1 + \frac{V_1}{2V_0} \sin \omega t \right]$$

This is the eqn of Velocity modulation

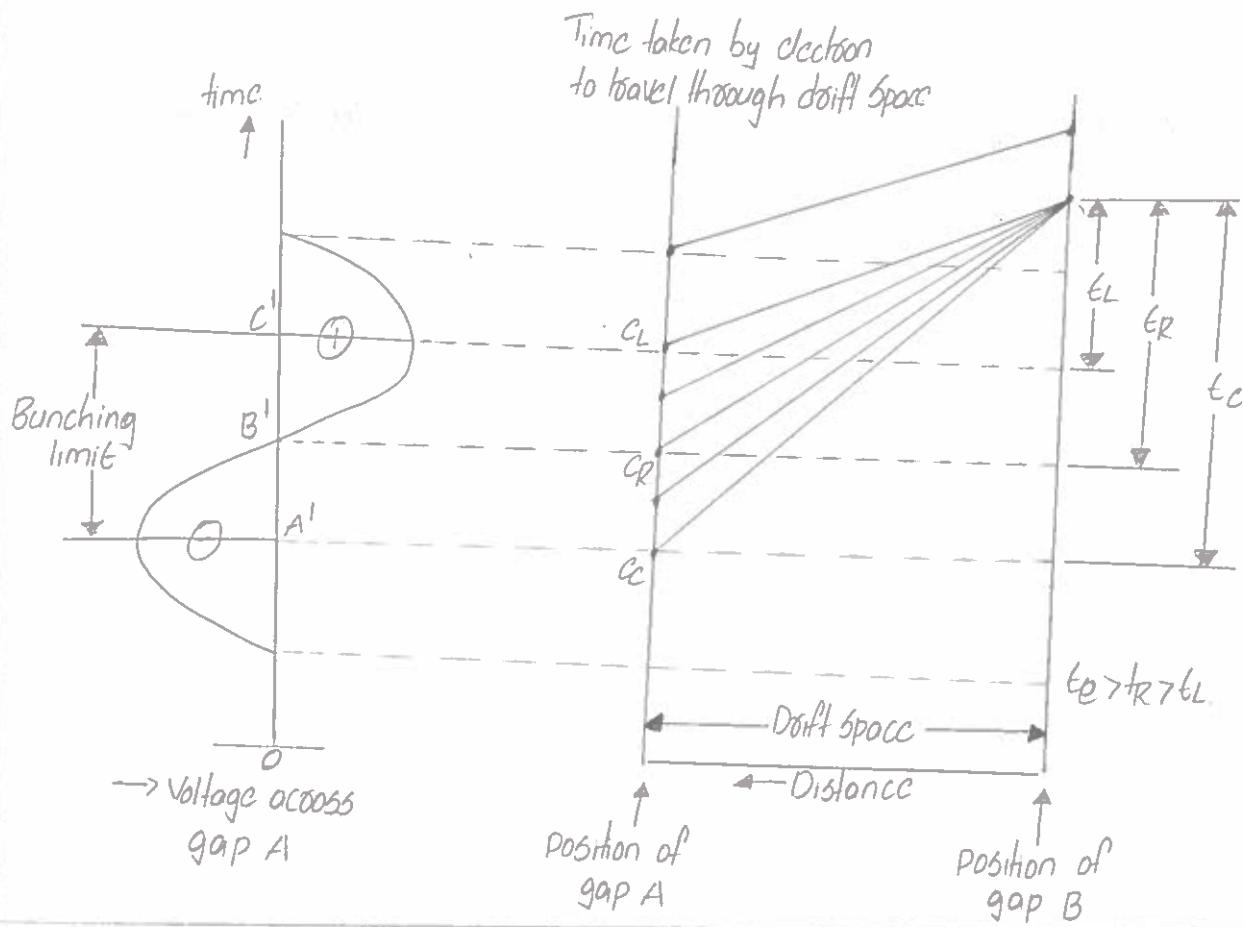
$$\theta_t = \theta_{t0} + \frac{\theta_0}{2}$$

Where θ_0 is the phase angle of the RF input Voltage during the which the electron is accelerated

$$\begin{aligned}\theta_0 &= \theta_t - \theta_{t0} = \theta(t) - \theta(t_0) \\ &= \frac{L_0 d}{V_0}\end{aligned}$$



Applegate diagram



- 1) The RF signal to be amplified is used for exciting the input buncher cavity thereby developing an alternating voltage of signal frequency across the gap A.
- 2) At point B on the input RF cycle, the alternating voltage is zero and going positive.
- 3) At this instant, the electric field across gap A is zero and an electron which passes through gap A at this instant is unaffected by the RF signal.
- 4) Let this electron be called the reference electron C_R which travels with an unchanged velocity $v_0 = \sqrt{2eV/m}$ where V is the anode to cathode voltage.
- 5) At point C of the input RF cycle an electron which leaves gap A later than reference electron C_R , called the late electron C_L is subjected to maximum positive RF voltage and hence travels towards gap B with an increased velocity and this electron tries to overtake the reference electron C_R .
- 6) Similarly an early electron C_E that passes the gap A slightly before the reference electron C_R is subjected to a maximum negative field.
- 7) Hence this early electron is decelerated and travels with a reduced velocity v_0 .
- 8) This electron C_E falls back and reference electron C_R catches up with the early electron.
- 9) Therefore, the velocity of electron varies in accordance with RF input voltage, resulting in velocity modulation of the electron beam.
- 10) The drift space converts the velocity modulation into current modulation.

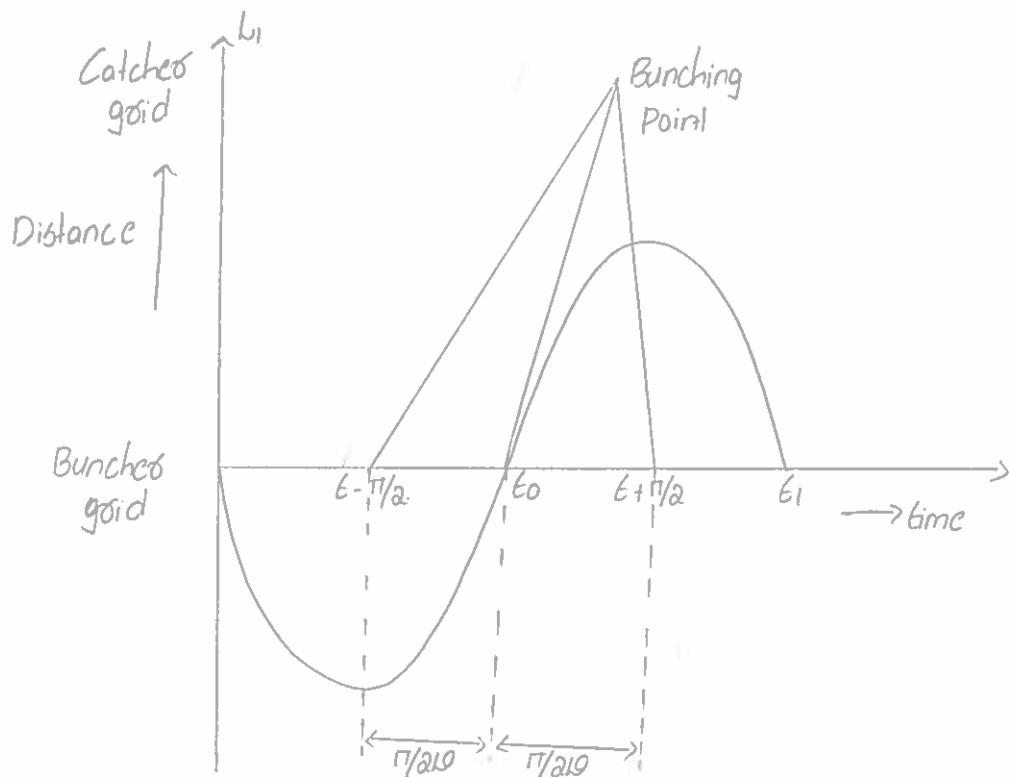
Bunching process of two cavity klystron Amplifier:

Maximum Velocity occurs at $+T/2$. So that

$$v_{(max)} = v_0 \left[1 + \frac{V_1}{2V_0} \right] \quad \textcircled{1}$$

Minimum Velocity at $-T/2$. So that

$$v_{(min)} = v_0 \left[1 - \frac{V_1}{2V_0} \right] \quad \textcircled{2}$$



If the distance in the drift space at which the bunching occurs from the bunched gold at time t is L_i .

$$L_i = v_0(t_i - t_0) \quad \text{--- (3)}$$

$$\text{The distance } L_i \text{ at } t - \pi/2\alpha = v_{\min}(t_i - t - \pi/2\alpha)$$

$$L_i \text{ at } t + \pi/2\alpha = v_{\max}(t_i - t + \pi/2\alpha) \quad \text{--- (4)}$$

$$\therefore t - \pi/2\alpha = t_0 - \pi/2\alpha \quad \text{--- (5)}$$

$$t + \pi/2\alpha = t_0 + \pi/2\alpha \quad \text{--- (6)}$$

From eqn (1), (2), (4) and eqn (6) we get

$$L_i \text{ at } t - \pi/2\alpha = v_0 \left(1 - \frac{v_i}{2v_0}\right) (t_i - t_0 + \pi/2\alpha)$$

$$L_i \text{ at } t + \pi/2\alpha = v_0 \left(1 + \frac{v_i}{2v_0}\right) (t_i - t_0 - \pi/2\alpha)$$

For maximum value

$$L_i = v_0(t_i - t_0) + v_0 \left[\frac{\pi}{2\alpha} - \frac{v_i}{2v_0} (t_i - t_0) - \frac{v_i}{2v_0} \cdot \frac{\pi}{2\alpha} \right]$$

If the distance have to be same for $-\pi/2\alpha, 0, +\pi/2\alpha$ bunches L_i for all those should be equal to $v_0(t_i - t_0)$; i.e;

$$\frac{\pi}{2\tau_0} - \frac{V_1}{2V_0} (t_1 - t_0) - \frac{V_1}{2\tau_0} \cdot \frac{\pi}{2\tau_0} = 0$$

$$- (t_1 - t_0) = \frac{\pi}{2\tau_0} \left(\frac{V_1}{2\tau_0} - 1 \right) \frac{2\tau_0}{V_1}$$

$$= \frac{\pi}{2\tau_0} - \frac{\pi}{\tau_0} \cdot \frac{V_0}{V_1}$$

has $\frac{V_0}{V_1}$ is very high, $\frac{\pi}{2\tau_0}$ can be neglected.

$$t_1 - t_0 = \frac{\pi}{\tau_0} \cdot \frac{V_0}{V_1}$$

Sub this eqn in eqn ③

$$L_1 = 18 \left(\frac{\pi}{\tau_0} \cdot \frac{V_0}{V_1} \right)$$

Bunching occurs as the RF signal changes from $-\pi/2$ to $\pi/2$ i.e. $\pm \pi/2$ for a value of $\pi = 3.14$ optimum bunching occurs

$$L_{max} = 3.14 \frac{V_0}{\tau_0} \cdot \frac{V_0}{V_1}$$

Expressions for output power and Efficiency:

Output power (P_{out}):

At the Catcher Cavity,

$$RF\ Voltage = V_2 \sin k_2 t_2$$

Energy given by the electron to the bunch

$$= (eC) V_2 \sin k_2 t_2 = -eV_2 \sin k_2 t_2$$

The average energy given to the RF field in a cycle

$$P_{av} = \frac{1}{2\pi} \int_{t_0}^{t_0 + L_0} (-eV_2 \sin k_2 t_2) dt_2 \quad \text{--- (1)}$$

In the Field Free Space b/w Cavities, the transit time for Velocity modulated electron is given by

$$T = t_2 - t_0 = \frac{L}{v_2} = \frac{L}{18 \left(1 + \frac{V_1}{2V_0} \right) \sin k_2 t_2}^{\frac{1}{2}}$$

$$= \frac{L}{18} \left[1 - \frac{V_1}{2V_0} \sin k_2 t_2 \right] \quad \text{--- (2)}$$

Multiplying by 10

$$10T = 10(t_2 - t_1) = \frac{LQ}{V_0} \left[1 - \frac{V_1}{2V_0} \sin(10t_1) \right] \quad (3)$$

In the above eqn, $\frac{LQ}{V_0} = T_0$, the transit time without RF Voltage V_1 in bunches Cavity and $\frac{LQ}{V_0} = 10T_0 = \theta_0 = 2\pi N$ is the transit angle without RF Voltage V_1 in bunches Cavity and N is the number of electron transit cycles in drift space.

The bunching parameter x of a klystron is defined by the eqn,

$$x = \frac{V_1}{2V_0} \theta_0 \quad (4)$$

which is a dimensionless quantity and proportional to input power.

eqn (1) can be written using eqn (3)

$$P_{av} = -\frac{CV_2}{2\pi} \int_0^{2\pi} \sin(10t_1 + \theta_0) dt_1$$

$$P_{av} = -\frac{CV_2}{2\pi} \int_0^{2\pi} \sin[\theta_0 + \theta_0 \left(1 - \frac{V_1}{2V_0} \sin(10t_1) \right)] dt_1$$

This is a bessel function and its solution is given by

$$P_{av} = -CV_2 J_1(x) \sin \theta_0 \quad (5)$$

Where, $J_1(x)$ = Bessel function of the first order for the argument x .

For N electron transit cycles,

$$\text{Energy transferred} = NP_{av} = -NeV_2 J_1(x) \sin \theta_0$$

$Ne = I_0$, the output current.

$$\text{Energy transferred} = -I_0 V_2 J_1(x) \sin \theta_0$$

Maximum value of $J_1(x) = 0.58$ for $x = 1.84$

For maximum energy transfer,

$$P_{max} = -I_0 V_2 (0.58) \sin \theta_0$$

$$\sin \theta_0 = -1, \theta_0 = 2\pi\pi - \pi/2$$

\therefore The output power, $P_{av} = P_{max} = 0.58 I_0 V_2$

(7)

Efficiency:

The input power is basically the dc input given by

$$P_{in} = I_0 V_0$$

The output power is

$$P_{out} = 0.58 I_0 V_0$$

The Efficiency is given by

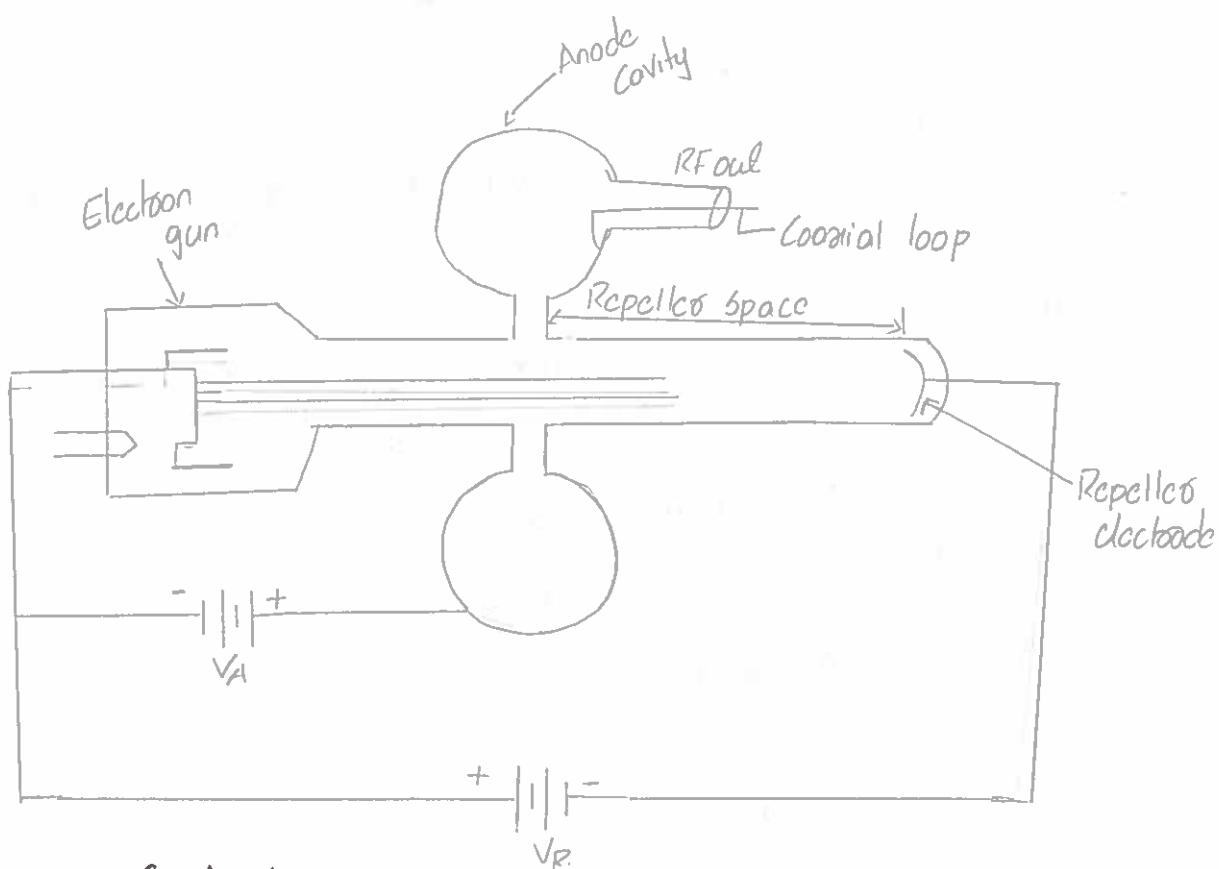
$$\eta = \frac{P_{out}}{P_{in}} = \frac{0.58 I_0 V_0}{I_0 V_0}$$

$$\eta = 0.58 \frac{V_0}{V_0}$$

As V_0 is always less than V_0 , the maximum efficiency that can be attained is 0.58 or 58%.

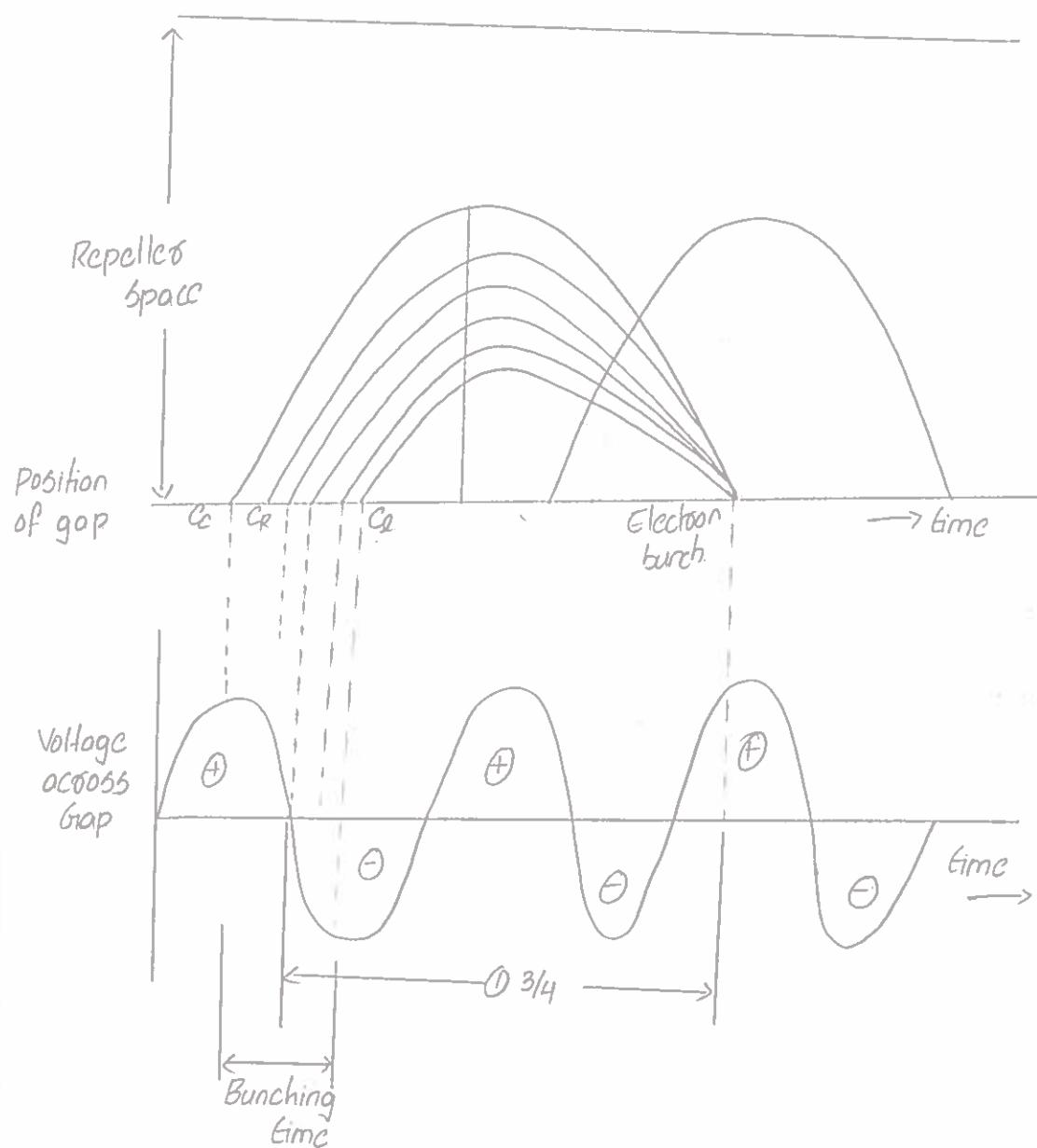
However practical efficiencies are only 15 to 30%. With CW powers of 500kW and pulsed powers of 30MW at 10GHz and power gain of 10-20 dB.

Reflex klystron:



Constructional details of Reflex klystron

- * It consists of an electron gun, a filament surrounded by cathode and a focussing electrode at cathode potential.
 - * The electron beam is accelerated towards the anode cavity.
 - * After passing the gap in the cavity, electrons travel towards a repeller electrode which is at a high negative potential V_R .
 - * The electrons never reach the repeller because of the negative field and are reflected back towards the gap.
 - * Under suitable conditions, the electrons give more energy to the gap than they took from the gap on their forward journey and oscillations are sustained.
- Applegate diagram



Applegate diagram.

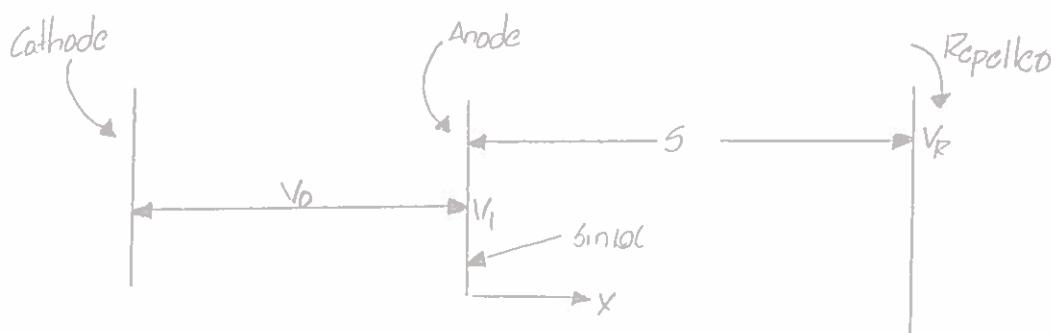
- * The RF Voltage that is produced across the gap by the Cavity oscillations act on the electron beam to cause Velocity modulation.
- * C_R is the reference electron that passes through the gap when the gap voltage is 0 and going negative. C_R is Unaffected by the gap Voltage.
- * This moves towards the repellos and gets reflected by the -ve Voltage on the repellos. It returns and passes through the gap for a second time.
- * The early electron C_E that passes through the gap before the reference electron C_R experiences a maximum +ve Voltage across the gap and this electron is accelerated.
- * It moves with greater Velocity and penetrates deep into repellos Space. Hence C_E and C_R appear at the gap for the second time at the same instant.
- * The late electron C_L that passes the gap later than reference electron C_R experiences a maximum -ve Voltage and moves with a retarding Velocity.
- * The return time is shorter as the penetration into repellos space is less and catches up with C_R and C_E electrons forming a bunch.
- * Bunches occur once per Cycle Centred around the reference electron C_R and these bunches transfer maximum energy to the gap to get sustained oscillations.
- * In general, the optimum transit time should be

$$T = n + \frac{3}{4}$$

Where n is an integer

This depends on repellos and anode Voltages.

Mathematical Theory of Bunching.



V_0 = electron gun anode Voltage

$V_1 \sin \theta$ = RF Voltage at Cavity gap

V_R = Repeller Voltage With respect to Cathode

s = Distance b/w Cavity gap and repeller electrode

v_0 = Velocity of electron in gun

v_i = Velocity due to RF Voltage in addition to the electron accelerating Voltage V_0

t_0 = Time for electron entering Cavity gap at $x=0$

t_1 = Time for same electron leaving Cavity gap at $x=d$

t_2 = Time for same electron retarded by retarding field at $x=d$.

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad \textcircled{1}$$

$$v_i = v_0 \sqrt{\frac{1+V_1}{V_0}} \sin \omega t \quad \textcircled{2}$$

Voltage now is $V_0 + V_1 \sin \omega t$, $V_1 \ll V_0$

Voltage b/w repeller and anode = $V_R - (V_0 + V_1 \sin \omega t) \approx V_R - V_0$

Retarding electrostatic field b/w repeller and anode is given by

$$E = -\left(\frac{V_R - V_0}{s}\right) \quad \textcircled{3}$$

Force on electron = $-cE = c\left(\frac{V_R - V_0}{s}\right)$

Also, Force on electrons = mass \times acceleration = $\frac{md^2x}{dt^2}$ $\textcircled{4}$

equating eqn $\textcircled{4}$ & eqn $\textcircled{5}$

$$\frac{md^2x}{dt^2} = \frac{c}{s}(V_R - V_0)$$

$$\frac{d^2x}{dt^2} = \frac{c}{ms}(V_R - V_0) \quad \textcircled{6}$$

Integrating eqn $\textcircled{6}$ once,

$$\frac{dx}{dt} = \frac{c}{ms}(V_R - V_0)t + C \quad \textcircled{7}$$

At $t=G$

$$\frac{dx}{dt} = v_i$$

$$v_i = \frac{c}{ms}(V_R - V_0)G + C$$

$$C = v_i - \frac{c}{ms}(V_R - V_0)G$$

Substituting for c in eqn ⑦, we get

$$\frac{da}{dt} = \frac{c}{m_s} (V_R - V_0)(t - t_1) + \vartheta_i \quad \text{--- } ⑧$$

Integrating eqn ⑧ once again

$$a = \frac{c}{2m_s} (V_R - V_0)(t - t_1)^2 + \vartheta_i(t - t_1) + C_1 \quad \text{--- } ⑨$$

At $a=0$ i.e. at the point of return from escape space; $t = t_2$

$$0 = \frac{c}{2m_s} (V_R - V_0)(t_2 - t_1)^2 + \vartheta_i t_2 + C_1$$

$$C_1 = -\frac{c}{2m_s} (V_R - V_0)(t_2 - t_1)^2 - \vartheta_i t_2.$$

Using this value of C_1 in eqn ⑨, we get

$$a = \frac{c}{2m_s} (V_R - V_0) [(t - t_1)^2 - (t_2 - t_1)^2] + \vartheta_i(t - t_2)$$

Again when $t = t_1$, $a = 0$;

$$-\frac{c}{2m_s} (V_R - V_0)(t_2 - t_1)^2 - \vartheta_i(t_2 - t_1) = 0$$

$(t_2 - t_1)$ is the round trip transit time and is given by

$$(t_2 - t_1) = \frac{-2m_s \vartheta_i}{c(V_R - V_0)} \quad \text{--- } ⑩$$

The transit angle \log is defined as transit angle at time t .

$$\log(t_2 - t_1) = \frac{-2m_s \vartheta_i}{c(V_R - V_0)}$$

$$\log_{t_2} = \log_{t_1} - \frac{2m_s \vartheta_i}{c(V_R - V_0)} \quad \text{--- } ⑪$$

From eqn ⑩

$$\vartheta_i = \vartheta_0 \left[1 + \frac{V_1}{V_0} \sin \log_{t_2} \right]^{\frac{1}{2}}$$

Since $V_1 \ll V_0$

$$\vartheta_i \approx \vartheta_0 \left[1 + \frac{V_1}{2V_0} \sin \log_{t_2} \right]$$

Substituting for V_1 in eqn ⑪

$$\log_{t_2} = \log_{t_1} - \frac{2m_s \vartheta_i}{c(V_R - V_0)} \cdot \vartheta_0 \left[1 + \frac{V_1}{2V_0} \sin \log_{t_2} \right] \quad \text{--- } ⑫$$

Let $\frac{-2m_s \vartheta_i \vartheta_0}{c(V_R - V_0)} = \log_{t_0}'$ $\log_{t_0}' \times \vartheta_0' = \vartheta_0'$

Where θ_0' is the round trip de transit angle of Centre of bunch electron.

$$\text{Let } \frac{V_1}{2V_0} \theta_0' = x' \quad (14)$$

Where x' is the bunching parameter.

Substituting in eqn (12)

$$\log_2 = \log_1 + \theta_0' \left[1 + \frac{V_1}{2V_0} \sin \theta_0 \right]$$

Power output:

$$P_{\text{out}} = \frac{2V_0 I_0 x' J_1(x')}{2\pi - \pi/2}$$

$$2\pi - \pi/2 = \theta_0' = \log_1' = \frac{2m\omega_0}{c(V_R - V_0)} \theta_0$$

The negative sign is not taken as electron bunch travels in a reverse direction, -x

$$P_{\text{out}} = \frac{2V_0 I_0 x' J_1(x')}{2m\omega_0 \theta_0} (V_R - V_0) c$$

Eliminating θ_0

$$P_{\text{out}} = \frac{2V_0 I_0 x' J_1(x')}{195} (V_R - V_0) \sqrt{\frac{c}{2mV_0}}$$

For maximum value of $x' J_1(x') = 1.75$, we get

$$P_{\text{max}} = \frac{125V_0 I_0 (V_R - V_0)}{195} \times \sqrt{\frac{c}{2mV_0}}$$

Efficiency:

DC power supplied by beam voltage V_0 is

$$P_{\text{dc}} = V_0 I_0$$

AC power delivered

$$P_{\text{ac}} = I_0 V_2 J_1(x') \sin \theta_0'$$

As the current flows in the negative direction, the negative sign becomes positive and $\sin \theta_0'$ is 1 and V_2 is V_1 being single and same cavity.

$$P_{\text{ac}} = I_0 V_1 J_1(x')$$

$$\text{Where } x' = \frac{V_1}{2V_0}$$

$$\frac{V_1}{V_0} = \frac{\alpha x'}{2n\pi - \pi/2}$$

$$P_{ac} = \frac{\alpha V_0 I_0 x' J_1(x')}{2n\pi - \pi/2}$$

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{\alpha V_0 I_0 x' J_1(x')}{V_0 I_0 (2n\pi - \pi/2)}$$

$$\mathcal{L} = \frac{\alpha x' J_1(x')}{2n\pi - \pi/2}$$

The factor $x' J_1(x')$ reaches a maximum value of 1.25 at $x' = 2.408$ and $J_1(x') = 0.52$. The maximum power output is obtained when $n=2$ or $\frac{1}{4}$ mode.

Maximum theoretical efficiency is

$$\mathcal{L} = \frac{\alpha(2.408)(0.52)}{2\pi(2) - \pi/2} \\ = 22.78\%$$

\therefore The practical values however are around 20%.

Oscillating modes and dip characteristics:

It consists of the klystron in parallel with beam loading Conductance G_{lb} , Cavity losses G_0 , lead inductance G_{lD} and parallel tuned Circuit.

The electronic admittance is given by

$$Y_C = \frac{I_0}{V_0}$$

Where I_0 and V_0 are output Current and Voltage Respectively.

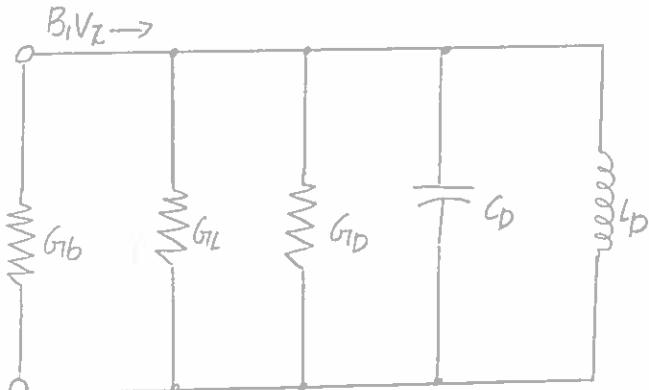
$$Y_C = \frac{\alpha I_0 J_1(x'_0) e^{-j\theta_0'}}{V_0 e^{j\pi/2}}$$

$$\text{Putting } V_1 = \frac{\alpha V_0 x'}{\theta_0}$$

$$Y_C = \frac{I_0 \cdot \theta_0'}{V_0 x'} e^{j(\pi/2 - \theta_0')}$$

Oscillations will take place when the net Conductance is less than zero

$$Y_C = G_C + jB_C$$



For oscillation, $|G_{rc}| \geq G_r$

$$\text{Where } G_r = \frac{1}{R_{sh}} = G_{ro} + G_{L} + G_{B}$$

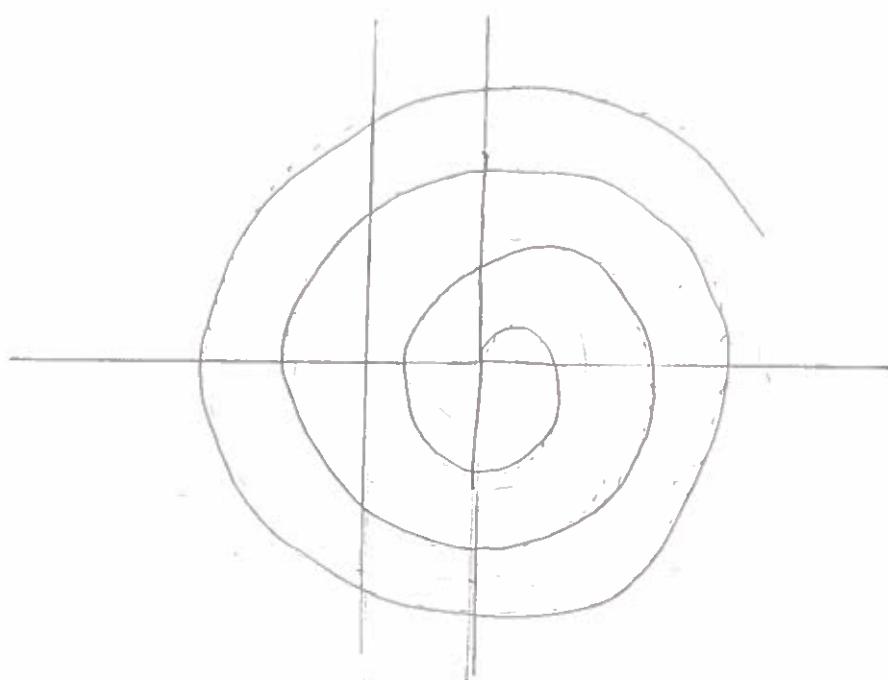
R_{sh} = Shunt Resistance

A rectangular plot of $G_r + jG_B$ will be a spiral. The electronic admittance Y_c is a function of transit angle θ_b' . Its phase is $\pi/2$ when θ_b' is θ .

$$n=1, \theta_b' = 2\pi n - \pi/2 = 3\pi/2; n=2, \theta_b' = 7\pi/2;$$

$$n=3, \theta_b' = 11\pi/2; n=4, \theta_b' = 15\pi/2.$$

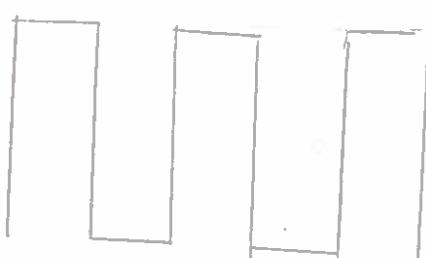
Practically the reflex klystron oscillations are possible, upto $n=7$.



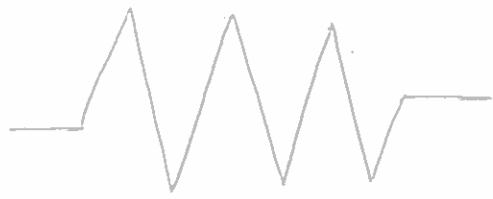
Slow Wave Structures:



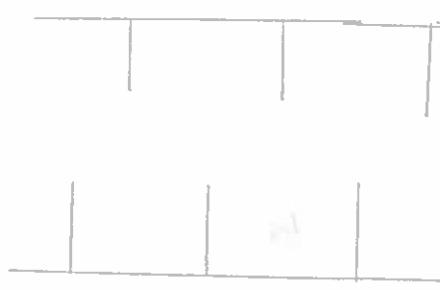
Helical Line



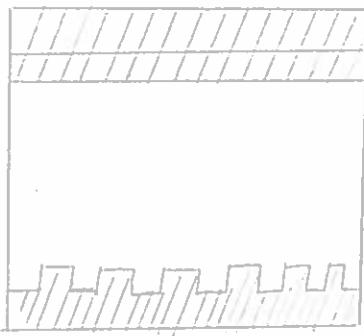
Folded back line.



Zig-Zag Line



Inter Digital Line



Corrugated Waveguide

* As the operating frequency is increased both the inductance & capacitance of resonant ckt must be decreased in order to maintain resonance at the operating frequency because the gain bandwidth product is limited by resonant ckt, the ordinary resonators can't generate a large op.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

* Several non resonant periodic ckt's (or) slow wave structures are designed for producing a large gain over a wide bandwidth.

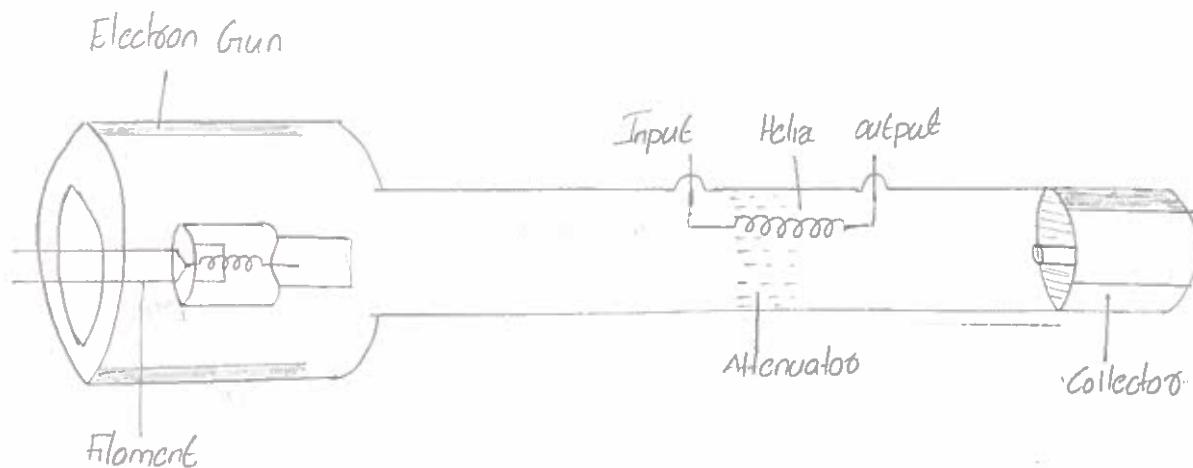
* Slow wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam & signal wave can interact.

* The phase velocity of a wave in ordinary waveguides is greater than velocity of light in vacuum.

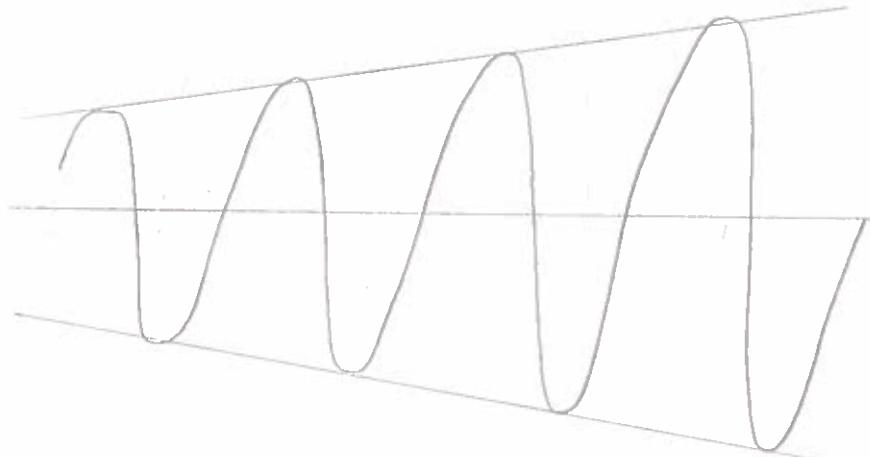
* In the operation of travelling wave & magnetron type devices the electron beam must keep in step with the microwave signal.

* Since the electron beam can be accelerated only to velocity's that are above a fraction of velocity of light, a slow wave structure might be incorporated in the microwave devices. The commonly used slow wave structure is Helical line.

Structure of TWT and Amplification process:



- * It has an electron gun as used in klystrons, which is used to produce a narrow Constant Velocity electron beam.
- * This electron beam is in turn passed through the centre of a long axial helix.
- * A magnetic focussing field is provided to prevent the beam from spreading and to guide it through the centre of helix.
- * Helix is loosely wound thin conducting helical wire, which acts as a slow wave structure.
- * The signal to be amplified is applied to the end of the helix adjacent to the electron gun. The amplified signal appears at the output or other end of helix under appropriate operating conditions.



- * When the applied RF Signal propagates around the turns of the helix, it produces an electric field at the centre of the helix. The RF field propagates with a Velocity of light.
- * The axial electric field due to RF Signal travels with Velocity of light multiplied by the ratio of helix pitch to helix Circumference.
- * When the Velocity of electron beam travelling through the helix approximates the rate of advance of the axial field, then interaction takes place b/w them in such a way that on an average the electron beam delivers energy to the RF Wave on the helix.
- * Thus, the Signal Wave grows and amplified output is obtained at the output of TWT. The axial Velocity v_p is given by

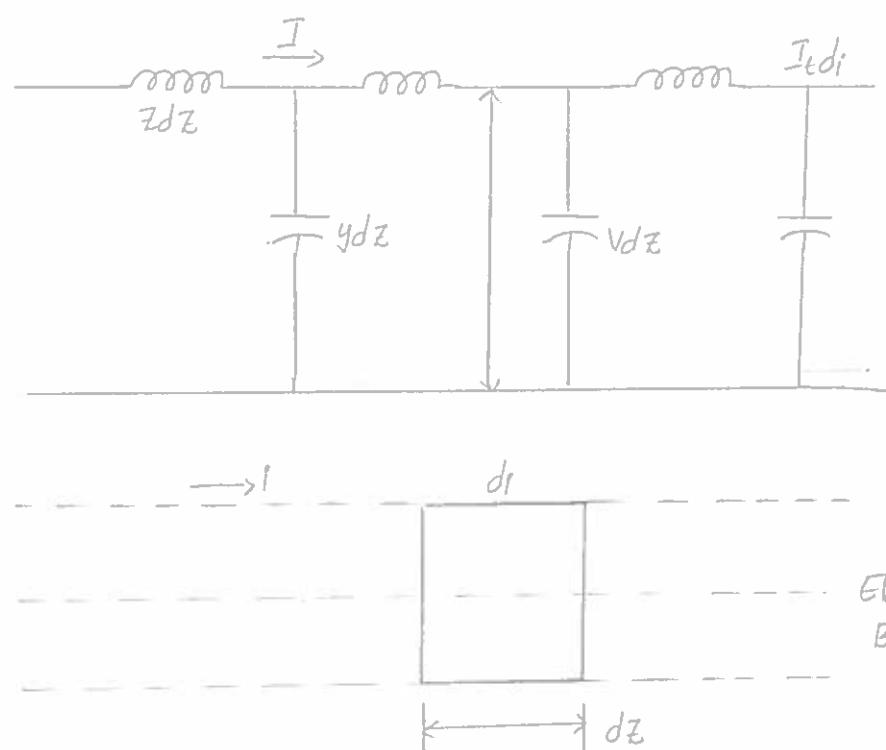
$$v_p = v_c \left(\frac{\text{Pitch}}{2\pi r} \right)$$

Where

r = Radius of helix

v_c = Velocity of light

Suppression of oscillations



$$dV = -IZdZ$$

$$dI = -VYdZ - di$$

Where $di = \left(\frac{di}{dZ}\right)dZ$ and i is the beam current $= \partial Z_1 \pi \theta_0^2$

We know, $\frac{d}{dZ} = -jB$ and relation b/w B and B_1 becomes

$$(B^2 - B_1^2)V = dBZ_1 = dBZ \partial Z_1 \pi \theta_0^2 \quad \text{--- (1)}$$

Voltage V is proportional to transverse component of electric field expressed in terms of ∂Z_1

Substitute for ∂Z_1 in eqn (1)

$$(B^2 - B_1^2) = \frac{-A}{(B - B_0)^2 - B_p^2}$$

Where $A = -2j\theta P_p^2 \times E_0 F_0 \pi \theta_0^2 \quad \text{--- (2)}$

A is a constant and $Z = jx$

Since eqn (2) is a fourth order eqn in B , it has four roots. Since $B_p \ll B_0$, there will be three forward wave solutions and one backward wave solution corresponding to $(B + B_1)$. If $(B + B_1)$ is replaced by $2B_1$.

Eqn (2) can be written as

$$(B - B_1)(B^2 - 2BB_0 + B_p^2) = -\frac{A}{2B_1} \quad \text{--- (3)}$$

$$B_1^2 = B_0^2 - B_p^2$$

$$(B - B_1)^3 = \frac{-A}{2B_1} \quad \text{--- (4)}$$

This eqn corresponds to those values of B .

$$B = B_1 + b(\cos \pi/3 + j \sin \pi/3)$$

$$B = B_1 + b(\cos \pi + j \sin \pi)$$

$$B = B_1 + b(\cos 5\pi/3 + j \sin 5\pi/3)$$

Where $b = \left(\frac{A}{2B_1}\right)^{1/3} \quad \text{--- (5)}$

$$G_t = -9.54 + 47.3 \text{ CN(dB)} \quad \text{--- (6)}$$

Where, c = gain parameter

$$= k \left(\frac{I_0}{V_0} \right)^{1/3}$$

N = Helia length in wavelength = π/λ_s

$$\lambda_s = v_p/f$$

k = Constant

v_p = axial phase Velocity

I_0 = dc beam Current

V_0 = dc beam Voltage

The gain will be maximum when the beam Velocity is approximately in synchronism with the axial phase Velocity v_p .

Illustration of problems.

- * A Four Cavity klystron has the following parameters. Beam Voltage $V_0 = 14.5 \text{ kV}$, Beam Current 1.4A, operating frequency 10 GHz, dc electron charge density 10^6 C/m^3 , RF charge density 10^8 C/m^3 , Velocity perturbations 10^5 m/sec . Compute the
- i) Dc electron Velocity
 - ii) The dc phase Current
 - iii) The plasma frequency
 - iv) The reduced plasma frequency for $R=0.4$.
 - v) The dc beam Current density
 - vi) The instantaneous beam Current density

Soln

$$\text{The dc electron Velocity } V_b = 0.593 \times 10^6 \sqrt{V_0}$$

$$= 0.593 \times 10^6 \sqrt{14.5 \times 10^3}$$

$$= 0.714 \times 10^8 \text{ m/sec.}$$

ii) The Dc phase Current = $\frac{I_0}{V_0} \frac{2\pi \times 10 \times 10^9}{0.714 \times 10^8}$
 $= 1.41 \times 10^8 \text{ Aod/sec.}$

The plasma frequency

$$\omega_p = \left[1.759 \times 10^{11} \times \frac{(\text{dc electron charge density})}{\epsilon_0} \right]^{1/2}$$

$$= \left[1.759 \times 10^{11} \times \left(\frac{10^6}{8.854 \times 10^{-12}} \right) \right]^{1/2}$$

$$\omega_p = 1.41 \times 10^8 \text{ rad/sec.}$$

The reduced plasma frequency ω_{pR} for $R=0.4$ is

$$\omega_{pR} = 0.4 \times \omega_p$$

$$= 0.41 \times 1.41 \times 10^8 \text{ rad/sec.}$$

$$\omega_{pR} = 0.564 \times 10^8 \text{ rad/sec.}$$

The dc beam current density

$$J_0 = P_0 / V_0$$

$$= 10^{-6} \times 0.714 \times 10^8 \text{ A/m}^2$$

$$J_0 = 71.4 \text{ A/m}^2$$

The instantaneous beam current density

$$J = 10^{-6} \times 0.714 \times 10^8 + 10^{-6} \times 10^5$$

$$J = 0.814 \text{ A/m}^2$$

- * A reflex klystron operates under the following conditions. $V_0 = 500 \text{ V}$, $R_{sh} = 20 \text{ k}\Omega$, $f_0 = 8 \text{ GHz}$, $L = 1 \text{ mm}$ is the spacing b/w repeller and cavity. The tube is oscillating at ω_0 at the peak on $n=2$ mode or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and through beam loading effect can be neglected.
- Find the value of repeller voltage V_R
 - Find the dc necessary to give microwave gap of voltage of 200 V .
 - Calculate the electronic efficiency.

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8} \cdot \frac{1}{L^2 C^2} \cdot \frac{C}{m} \left[2\pi n - \frac{\pi}{2} \right]^2$$

$$= \frac{1}{8} \cdot \frac{(1.759 \times 10^{11})(2\pi/2) - (\pi/2)^2}{(2\pi \times 8 \times 10^9)^2 (10^{-3})^2}$$

$$= 0.023$$

$$(V_R - V_0)^2 = 500 \times 0.023$$

$$= 11.5$$

$$V_R - V_0 = 11.5$$

$$V_R = 3.39 + 500$$

$$V_R = 503.39 \text{ Volts}$$

Assuming $B_0 = 1$,

$$V_i = R_{sh} \cdot I_2 = 2I_0 J_1(x') R_{sh}$$

$$I_0 = \frac{V_i}{2J_1(x') R_{sh}}$$

$$= \frac{200}{2 \times 0.582 \times 20 \times 10^3}$$

$$I_0 = 8.59 \times 10^{-3} \text{ A}$$

$$\text{Efficiency } \eta = \frac{2x' J_1(x')}{2\pi n - \pi/2}$$

$$x' = \frac{B_i V_i \theta_0'}{2V_0}$$

$$\theta_0' = \lg T_0' = \frac{\lg 2 \cdot m L V_0}{C(V_R - V_0)}$$

$$= \frac{2\pi \times 8 \times 10^9 \times 2 \times 10^{-3} \times 0.593 \times 10^6 \sqrt{500}}{1.579 \times 10^{11} \times [503 - (-500)]}$$

$$\theta_0' = 7.556$$

for $B_i = 1$,

$$x' = \frac{200 \times 7.556}{2 \times 500} = 1.51$$

$$x' = 1.51, \quad x' J_1(x') = 0.84$$

$$\eta = \frac{2 \times 0.84}{2\pi(2) - \pi/2}$$

$$\eta = 15.28\%$$

* A helical TWT has diameter of 2mm with 50 turns per cm.

a) Calculate axial phase Velocity

b) The anode Voltage at which the TWT has to be operated for useful gain

Soln:

$$v_p = \text{Velocity of light} \times \frac{\text{pitch}}{\text{circumference}}$$

$$\text{pitch} = \frac{1}{50\text{mm}} = 0.02\text{cm}$$

$$= 2 \times 10^{-4}\text{m}$$

$$\text{Circumference} = \pi D = \pi \times 2 \times 10^{-3}\text{m}$$

$$= 6.284 \times 10^{-3}\text{m}$$

$$v_p = 3 \times 10^8 \times 2 \times 10^{-4}$$

$$\frac{6.284 \times 10^{-3}}{}$$

$$v_p = 954 \times 10^6\text{ m}$$

$$CV_0 = \frac{1}{2} m v_p^2$$

$$V_0 = \frac{1}{2} \cdot \frac{m}{e} v_p^2$$

$$= \frac{1}{2} \times 9.1 \times \frac{10^{-31}}{1.6 \times 10^{-19}} \times (0.954 \times 10^7)^2$$

$$V_0 = 25.92\text{kV}$$

* An X-band pulsed cylindrical magnetron has $V_0 = 30\text{kV}$, $I_0 = 80\text{A}$, $B_0 = 0.01\text{ Wb}/6q\text{m}$, $a = 4\text{cm}$, $b = 8\text{cm}$. Calculate

a) Cyclotron angular frequency

b) Cut-off Voltage

c) Cut off magnetic flux density

Cyclotron angular frequency is given by

$$\omega = \frac{eB_0}{m}$$

$$\omega = \frac{1.6 \times 10^{-19} \times 0.01}{9.1 \times 10^{-31}}$$

$$\omega = 1.759 \times 10^9 \text{ rad/s}$$

Hull cut off Voltage is given by

$$V_{HC} = \frac{CB_0^2 b^2}{8m} \left[1 - \frac{a^2}{b^2} \right]^2$$

$$V_{HC} = \frac{1}{8} \times 1.759 \times 10^{11} \times (0.01)^2 \times (8 \times 10^{-2})^2 \times \left[1 - \frac{4^2}{8^2} \right]^2$$

$$V_{HC} = 7.9155 \text{ kV}$$

Cut off magnetic flux density is given by.

$$B_c = \frac{(8V_0 m/c)^{1/2}}{b(1-a^2/b^2)}$$

$$B_c = \frac{\left[8 \times 30 \times 10^3 \right]^{1/2}}{1.759 \times 10^{11}} \times \frac{1}{8 \times 10^{-2} \left[1 - \frac{4^2}{8^2} \right]}$$

$$B_c = \frac{1}{0.06} \times 0.001168$$

$$B_c = 19.468 \text{ mWb/m}^2$$

- * A Reflex klystron is operated at 56 Hz with an anode voltage of 600V and cavity gap 2mm. Calculate the gap transit angle. Find optimum length of the drift region. Assume $N = \frac{13}{4}$, $V_R = -500$ V.

Soln:

$$|V_R| = 6.74 \times 10^6 \times F \times \ln \frac{\sqrt{V_0}}{N} - V_0$$

$$500 = \frac{6.74 \times 10^6 F \ln \sqrt{V_0} - V_0}{N}$$

$L = 2.463 \text{ mm}$ (length of drift region)

Also gap transit angle $\theta_g = \log \frac{U_f}{U_0}$

$$d = 2 \times 10^{-3} \text{ m},$$

$$U_0 = 5.93 \times 10^5 \sqrt{V_0}$$

$$= 18.75 \times 10^6 \text{ m/s}$$

$$\omega = 5 \times 10^9 \text{ rad/s}$$

$$\text{Transit angle, } \theta_g = \frac{2\pi \times 5 \times 10^9 \times 2 \times 10^{-3}}{18.75 \times 10^6}$$

$$= 3.351 \text{ radians}$$

- * A two Cavity Klystron is operated at 10 GHz with $V_0 = 1200 \text{ V}$, $I_0 = 30 \text{ mA}$, $d = 1 \text{ mm}$, $L = 4 \text{ cm}$ and $R_{sh} = 40 \text{ k}\Omega$. Neglecting beam loading calculate.
- input RF Voltage V_i for a maximum output voltage.
 - voltage gain
 - Efficiency

Soln

The bunching parameter X is given by

$$X = \frac{V_i}{2V_0} \theta_0$$

Where, θ_0 = transit angle without RF Voltage

$$\theta_0 = \frac{L}{V_0}$$

V_0 = Velocity of defocence electron

$$V_0 = 0.593 \times 10^6 \sqrt{V_0}$$

$$= 20.54 \times 10^6 \text{ m/s}$$

$$\theta_0 = \frac{2\pi \times 10^9 \times H \times 10^{-2}}{20.54 \times 10^6} = 122.347 \text{ rad}$$

$$V_i = \frac{2XV_0}{\theta_0}$$

Now for maximum of power

$$X = 1.84$$

$$(V_i)_{max} = \frac{2 \times 1.84 \times 1200}{122.347}$$

$$= 36.09 \text{ V}$$

If beam Coupling Coefficient is Considered.

$$V_i = \frac{2XV_0}{B_i \theta_0}$$

$$\text{Where } B_i = \frac{\sin \theta_{g/2}}{\theta_{g/2}}$$

$$\theta_g = \text{average gap transit angle} = \frac{12d}{V_0}$$

$$C_g = \frac{122.347}{4 \times 10^2} \times 10^{-3} = 3.05 \text{ pF}$$

$$B_i = \frac{\sin 1.5293}{1.5293} = 0.653$$

$$(V_1)_{\max} = \frac{36.09}{0.653}$$

$$(V_1)_{\max} = 55.268 \text{ V}$$

Voltage gain A_V is given by

$$A_V = \frac{V_2}{V_1}$$

Where $V_2 = B_0 I_2 R_{sh}$

B_0 = d/p Cavity Coupling Coefficient = B_i

$$I_2 = 2 I_0 J_1(X)$$

$$J_0(X) = 1.84$$

$$J_1(X) = 0.58$$

$$I_2 = 2 \times 30 \times 10^{-3} \times 0.58$$

$$V_2 = 0.653 \times 2 \times 30 \times 10^{-3} \times 0.58 \times 40 \times 10^3$$

$$V_2 = 909.49 \text{ V}$$

$$A_V = \frac{V_2}{V_1} = \frac{909.49}{55.268} = 16.45$$

$$A_V = 24.33 \text{ dB}$$

Maximum efficiency is given by

$$\eta = 0.58 \times \frac{V_2}{V_0}$$

$$= \frac{0.58 \times 909.49}{1200}$$

$$\eta = 43.95\%$$

- * A Reflex klystron operates at 8GHz at the peak of $n=2$ mode $V_0=300\text{V}$, $R_{sh}=20\text{k}\Omega$, and $L=1\text{mm}$. If the gap transit time and beam loading are neglected. Find the
- Repeller Voltage

- beam Current necessary to obtain an RF gap Voltage of 200V.

Soln: Repeller Voltage V_R is given by

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8} \cdot \frac{1}{L^2 C} \frac{c}{m} \left[2\pi n - \frac{\pi}{2} \right]^2$$

$$= \frac{1}{8} \times \frac{(1759 \times 10^{11}) \times (2\pi \times 2 - \frac{\pi}{2})^2}{(2\pi \times 8 \times 10^9)^2 \times (10^{-3})^2}$$

$$\frac{V_0}{(V_R - V_0)^2} = 0.00105$$

$$(V_R - V_0)^2 = \frac{300}{0.00105}$$

$$= 285 \times 1.34 \times 10^3$$

$$V_R - V_0 = 533.98 \text{ V}$$

$$V_R = 533.98 + V_0$$

$$V_R = 833.98 \text{ V}$$

Assuming output Coupling Coefficient $B_0 = 1$

$$V_i = I_a \cdot R_{sh} = 2I_a J_1(x') R_{sh}$$

$$I_a = \frac{V_i}{2J_1(x') R_{sh}}$$

$$= \frac{200}{2 \times 0.582 \times 20 \times 10^3}$$

$$I_a = 8.59 \text{ mA}$$

* The parameters of a two cavity amplifying klystron are $V_0 = 1200 \text{ V}$, $I_a = 28 \text{ mA}$, $f = 8 \text{ GHz}$, gap spacing in either cavity $d = 1 \text{ mm}$, spacing b/w two cavities $s: L = 4 \text{ cm}$, effective shunt resistance $R_{sh} = 40 \text{ k}\Omega$.

- Find the input microwave voltage V_i in order to generate maximum d/p voltage
- Determine the Voltage gain
- Calculate the efficiency of the amplifier neglecting beam loading
- Compute the beam loading Conductance and show that one may neglect it in

the preceding Calculations.

Soln

$$\left(\frac{V_1}{V_0}\right)_{max} = \frac{3.68}{2\pi n - \pi/2}$$

$2\pi n - \pi/2 = \theta_0$ = transit angle b/w cavities

$$= \frac{19.6}{190}$$

$$= \frac{2\pi \times 8 \times 10^9 \times 10^{-2} \times 4}{0.593 \times 10^6 \sqrt{1200}}$$

$$= 97.88$$

$$(V_p)_{max} = \frac{1200 \times 3.68 \times 0.593 \times 10^6 \sqrt{1200}}{2\pi \times 8 \times 10^9 \times 10^{-2} \times 4}$$

$$(V_p)_{max} = 45 \text{ Volts.}$$

If beam amplifying Coefficient B_i is Considered.

$$B_i = B_0 = \frac{\sin(\theta_0/2)}{\theta_0/2}$$

$$\theta_0 = \frac{kq}{190}$$

$$= \frac{2\pi \times 8 \times 10^9 \times 10^{-3}}{0.593 \times 10^6 \sqrt{1200}}$$

$$\theta_0 = 2.45 \text{ rad}$$

$$B_i = \frac{\sin(2.45)}{2.45}$$

$$B_i = 0.768$$

$$(V_i)_{max} = \frac{45}{0.768} = 58.59 \text{ Volts}$$

In our derivation in text, $B_i = B_0 = 1$ has been assumed.

Voltage gain of klystron amplifier

$$A_V = \frac{V_2}{V_1} = \frac{B_0 J_2 R_{sh}}{V_1}$$

$$V_1 = \frac{2V_0 x}{B_0 \theta_0}$$

$$I_2 = 2I_0 J_1(x)$$

If the beam Coupling Coefficient of bunches B_i and Catcher B_0 are equal
i.e; $B_i = B_0$

$$A_V = \frac{B_0 \cdot 2I_0 J_1(x)}{CV_0 x} B_0 \theta_0 R_{sh.}$$

$$= \frac{B_0^2}{R_0} \cdot \frac{J_1(x)}{x} \theta_0 R_{sh.}$$

R_c = Resistance of Wall of Catcher Cavity

R_B = Beam loading resistance.

R_L = Elettron load resistance.

$$A_V = \frac{(0.768)^2 \times 97.88 \times 0.582 \times 40 \times 10^3}{1200}$$

$$= \frac{28 \times 10^{-3} \times 1.841}{28 \times 10^{-3} \times 1.841}$$

$$J_1(x) = 0.582$$

$x = 1.841$ for maximum V_2 .

$$A_V = 17.034$$

$$\text{Efficiency} = \eta = 0.58 \times \frac{V_2}{V_0}$$

$$V_2 = B_0 I_2 R_{sh.}$$

$$B_0 I_2 = 2 \times 28 \times 10^{-3} \times 0.582 \times 0.768 \times 40 \times 10^3$$

$$= 1001.23$$

$$\eta = 0.58 \times \frac{1001.23}{1200}$$

$$\eta = 48.39\%$$

Beam loading Conductance

$$G_B = \frac{G_0}{2} \left(B_0^2 - B_0 \cos \theta_0 \frac{\partial g}{\partial x} \right)$$

$$= 23.3 \times 10^{-6} \left[(0.768)^2 - (0.768) \cos(2.45) \right]$$

$$R_B = \frac{1}{G_B} = 0.073 \times 10^6$$

$$R_B = 73 \text{ k}\Omega$$

* A two cavity klystron amplifier has the following characteristics. Voltage gain 15dB, input power 6mW, R_{sh} of input cavity $30\text{k}\Omega$, R_{sh} of output cavity $10\text{k}\Omega$, $R_L \cdot 40\text{k}\Omega$. Determine

- The input rms Voltage
- The output rms Voltage
- The power delivered to the load.

Soln.

$$P_{in} = \frac{V_i^2}{R}$$

$$= P_{in} \times R_{sh}$$

$$= 5 \times 10^3 \times 30 \times 10^3 = 150$$

$$V_i = 12.25 \text{ V.}$$

$$AV = 20 \log \frac{V_2}{V_i} \text{ dB}$$

$$15 = 20 \log \frac{V_2}{12.25}$$

$$V_2 = 68.89 \text{ V.}$$

$$P_{out} = \frac{V_2^2}{R_{sh}}$$

$$= \frac{(68.89)^2}{20 \times 10^3}$$

$$= \frac{47145.83}{20 \times 10^3}$$

$$= 0.2373 \text{ W}$$

$$P_{out} = 237.3 \text{ mW}$$